Radiative Interchange between Semi-Infinite Parallel Strips: A Simple Two-Dimensional Method

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Theme

A SIMPLE and straightforward technique of multiple numerical quadrature is used to investigate two-dimensional radiative transfer between semi-infinite parallel strips. This novel technique permits the dimension in the infinite direction to be divided into any number of intervals to facilitate the numerical integration. For the work herein discussed, 3 intervals are sufficient to yield adequate solutions.

Content

There are a large number of articles in the literature that deal with the problem of radiative exchange in variously shaped, evacuated enclosures. A survey of these articles is available in Ref. 1.

In this investigation, Gaussian quadrature techniques are applied to the integral equation that describes a two-dimensional, parallel plate, radiative transfer problem. This elementary geometry is anlyzed by applying the Gauss-Legendre quadrature directly to the finite ($\bar{x}$) segment and somewhat indirectly to the semi-infinite ($\bar{y}$) segment. Application of physical reasoning readies the resulting governing equations for easily obtained solutions. Although various numerical quadrature techniques are applicable, this is the only one that is successful. Only the dimensionless radiosity is considered, because all other quantities, such as the local heat transfer, can be determined directly from this quantity.

It is assumed that both plates are two-dimensional, gray, diffusely reflecting and emitting, constant temperature, semi-infinite, parallel strips separated by a nonparticipating medium (Fig. 1). For this case, the governing equation for the dimensionless radiosity is

$$B(x_1,y_1)=1+\frac{\rho h^2}{\pi}\int_0^C\int_0^\infty \frac{B(x_2,y_2)\,dx_2dy_2}{[(x_2-x_1)^2+(y_2-y_1)^2+h^2]^2} \quad (1)$$

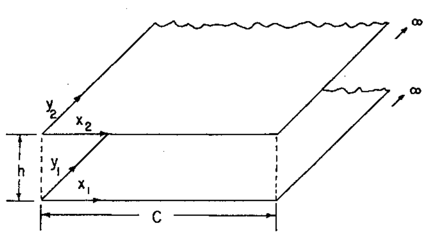

Fig. 1 Semi-infinite parallel strips geometry.

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The first step toward a solution is to normalize the position variables with respect to the separation distance h and break the y_2 integration (in the infinite direction) into $n+1$ parts. If, in addition, the nondimensional quantity (S/h) is chosen large enough such that for all y greater than S the radiosity varies in the x-direction only, Eq. (1) can be transformed to

$$B(h\bar{x}_1,h\bar{y}_1)=1+\frac{\rho}{\pi}\int_0^{C/h}\int_0^{S_1/h}\frac{B(h\bar{x}_2,h\bar{y}_2)\,d\bar{y}_2d\bar{x}_2}{[(\bar{x}_2-\bar{x}_1)^2+(\bar{y}_2-\bar{y}_1)^2+1]^2}+\ldots$$
$$+\frac{\rho}{\pi}\int_0^{C/h}\int_{S_n/h}^{S/h}\frac{B(h\bar{x}_2,h\bar{y}_2)\,d\bar{y}_2d\bar{x}_2}{[(\bar{x}_2-\bar{x}_1)^2+(\bar{y}_2-\bar{y}_1)^2+1]^2}$$
$$+\frac{\rho}{\pi}\int_0^{C/h}B(h\bar{x}_2,S)f(\bar{x}_1,\bar{y}_1,\bar{x}_2,S/h)\,d\bar{x}_2 \quad (2)$$

in which $f(\bar{x}_1,\bar{y}_1,\bar{x}_2,S/h)$ is the configuration factor of the point $(\bar{x}_1,\bar{y}_1)$ on the infinitesimal strip extending from $\bar{y}_2=S/h$ to $\bar{y}=\infty$ and $\bar{x}=\bar{x}_2$. The factor is defined by

$$f(\bar{x}_1,\bar{y}_1,\bar{x}_2,S/h)=\int_{S/h}^\infty\frac{d\bar{y}_2}{[\bar{x}_2-\bar{x}_1)^2+(\bar{y}_2-y_1)^2+1]^2}$$
$$=\frac{\pi}{4[(\bar{x}_2-x_1)^2+1]^{3/2}}$$
$$-\frac{S/h-\bar{y}_1}{2[(\bar{x}_2-\bar{x}_1)^2+1][(S/h-\bar{y}_1)^2+(\bar{x}_2-\bar{x}_1)^2+1]^2}$$
$$-\frac{1}{2[(\bar{x}_2-\bar{x}_1)^2+1]^{3/2}}\tan^{-1}[\frac{S/h-\bar{y}_1}{((\bar{x}_2-\bar{x}_1)^2+1)^{1/2}} \quad (3)$$

After approximating the integrals with the Gauss-Legendre quadrature, the system of (m) times $(n+1)$ equations in (m) times $(n+1)$ unknowns can be represented by

$$B_{kl}=1+\frac{\rho}{\pi}\sum_{i=1}^{m}\sum_{j=1}^{n1}\frac{B_{ij}W_iW_j}{[(\bar{x}_i-\bar{x}_k)^2+(\bar{y}_j-\bar{y}_l)^2+1]^2}+\ldots$$
$$+\frac{\rho}{\pi}\sum_{i=1}^{m}\sum_{j=np}^{n}\frac{B_{ij}W_iW_j}{[(\bar{x}_i-\bar{x}_k)^2+(\bar{y}_j-\bar{y}_l)^2+1]^2}$$
$$+\frac{\rho}{\pi}\sum_{i=1}^{m}B_{i(m+1)}f(\bar{x}_k,\bar{y}_l,\bar{x}_i,S/h)\,W_i \quad (4)$$

To determine the first parameter, S/h, a two-interval approximation of the method is used. Figure 2 is a sketch of the resulting computed radiosity along the centerline ($\epsilon=0.9$, $C/h=1.0$, and $n=m=5$) of the plate for various values of S/h. Experience indicates that these values of ϵ and C/h yield easily obtained solutions. For $S/h=1$, the radiosity distribution has a discontinuous slope at S/h where the radiosity is 1.0420. This discontinuity is an indication that S/h should be larger. For $S/h=3$, the slope is continuous, and the radiosity levels out at a value of 1.0462. This is a good

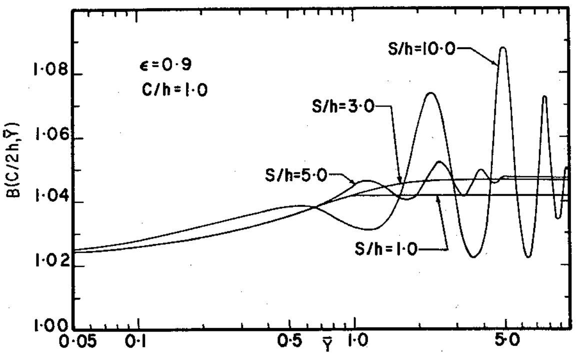

Fig. 2 Effect of varying S/h on convergence at the centerline; $(m, n) = (5,5)$.

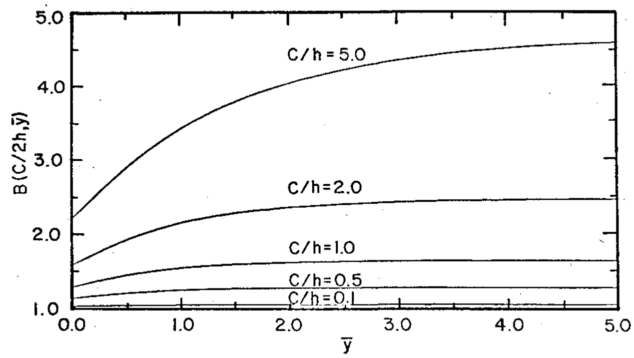

Fig. 3 $B(C/2h,\bar{y})$ for $\epsilon=0.10$, $(m,n)=(10,10)$, $S_l/h=2$, and $S/h=5$; ($\bar{y}=0$ is the edge).

Table 1 Comparison of $C/h=5$ solution at $(\bar{x}/C/h)=0.5$ with one-dimensional solution

$\bar{y}$	$\epsilon=0.9$		$\epsilon=0.1$	
	1-*D*	2-*D*	1-*D*	2-*D*
0	1.05409	1.04978	3.16228	2.21698
0.5	1.07861	1.07349	4.30835	2.89598
1.0	1.09327	1.08743	5.27780	3.42474
2.0	1.10440	1.09746	6.63256	4.04953
3.0	1.10781	1.10014	7.51627	4.36310
4.0	1.10919	1.10108	8.12604	4.52514
5.0	1.10987	1.10149	8.56122	4.60190
∞	1.11111		10.00000	

solution, because the one-dimensional limiting value at the centerline for these same variables is 1.0467. For $\bar{y}$ less than 1, the solution coincides with the solution for $S/h=1$. When S/h has the values of 5 and 10, maxima of 1.0522 and 1.0871 occur at $\bar{y}$ at approximately 2.5 and 5, respectively. There is no way to justify local maxima of this nature. Thus, when using the two-interval approximation, if S/h is too large, an

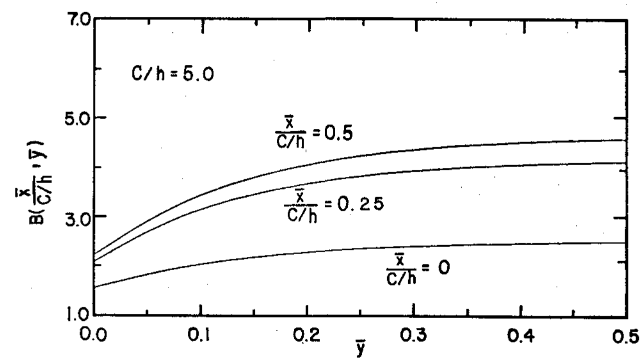

Fig. 4 $B(\bar{x}h/C\bar{y})$ for $\epsilon=0.1$, $(m,n)=(10,10)$, $S_l/h=2$ and $S/h=5$, ($\bar{x}h/C=0.5$ is the center).

oscillation is encountered in the solution with a maximum occurring at $\bar{y}=S/2h$. With S/h set at a particular value, the effects of variations in S_l/h, ϵ, C/h, and the orders of quadrature (both $\bar{x}$ and $\bar{y}$) can be investigated. To increase the S/h value and still have acceptable solutions, the orders of quadrature must be increased. For example, for the same condition as presented for Fig. 2, satisfactory solutions may be obtained for $S/h=5$ if the orders of quadrature are increased to $n=m=10$. A complete discussion is to be found in the backup paper.

Figure 3 is an example of a typical solution and is presented to illustrate the variation in the dimensionless radiosity along the centerline of the plates when C/h is varied. As C/h becomes large, the solution at the centerline ($\bar{x}h/C=0.5$) approaches the one-dimensional solution. (In fact, at $\bar{y}=S/h$, it is a maximum and should approach $1/\epsilon$ as C/h approaches infinity.) The one-dimensional problem has been presented by Sawheny.[2] Table 1 may be used to compare the one-dimensional solution of Sawheny to the two-dimensional ($C/h=5$) solution of the present study at various depths along the centerline for $\epsilon=0.9$ and $\epsilon=0.1$. The table reveals that the one-dimensional solution is greater at every point than the two-dimensional solution for $C/h=5$ (the limiting value of C/h used in the present study). For the case of $\epsilon=0.9$, the values essentially agree, whereas the values resulting from the $\epsilon=0.1$ situation do not but are appropriately bounded.

Figure 4 illustrates how the radiosity varies along the plates at three different values of $\bar{x}$. This plot indicates that the radiosity changes much more between 0 and 0.25 than between 0.25 and the centerline ($\bar{x}h/C=0.5$). In addition, the slope at the edge ($\bar{y}=0$) is less for smaller values of $\bar{x}h/c$.

References

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[2]Sawheny, T.R., "Radiant Interchange in a Non-Isothermal Rectangular Cavity," Master's thesis, Mechanical and Aerospace Engineering Department, University of Missouri, Rolla, Mo., 1971.